C1 January 2013 (MA)

Q1) $x - 4x^3 = x(1-4x^2)$

$= \boxed{x(1-2x)(1+2x)}$

Q2) $8^{2x+3} = 2^{3(2x+3)}$

$= 2^{6x+9}$

$\therefore \boxed{y = 6x+9}$

Q3i) $(5-\sqrt{8})(1+\sqrt{2}) = 5+5\sqrt{2} - \sqrt{8} - \sqrt{8}\sqrt{2}$

$= 5+5\sqrt{2} - \sqrt{4}\sqrt{2} - \sqrt{4}\sqrt{2}\sqrt{2}$

$= 5+5\sqrt{2} - 2\sqrt{2} - 4$

$= \boxed{1+3\sqrt{2}}$

ii) $\sqrt{80} + \dfrac{30}{\sqrt{5}} = \sqrt{80} + \dfrac{30}{\sqrt{5}}\left(\dfrac{\sqrt{5}}{\sqrt{5}}\right)$

$= \sqrt{80} + \dfrac{30\sqrt{5}}{5}$

$= \sqrt{16}\sqrt{5} + 6\sqrt{5}$

$= 4\sqrt{5} + 6\sqrt{5}$

$= \boxed{10\sqrt{5}}$

Q4) $u_{n+1} = 2u_n - 1, n \geq 1$

a) Given $u_2 = 9$,

$u_3 = 2u_2 - 1$

$u_3 = 2(9) - 1$

$\boxed{u_3 = 17}$

$u_4 = 2u_3 - 1$

$u_4 = 2(17) - 1$

$\boxed{u_4 = 33}$

b) $\sum_{r=1}^{4} u_r = u_1 + u_2 + u_3 + u_4$

$= u_1 + 9 + 17 + 33$

$= u_1 + 59$

Since $u_2 = 9$, $u_2 = 2u_1 - 1$

$9 = 2u_1 - 1$

$u_1 = 5$

$\therefore \sum_{r=1}^{4} u_r = 5 + 59$

$\boxed{= 64}$

Q5) $l_1 : y = -2x + 3$

l_2 : perpendicular to l_1, passes through $(5, 6)$

Gradient of $l_1 = -2$

$\therefore$ Gradient, m of $l_2 = \frac{1}{2}$.

Equation of l_2 : $y - y_1 = m(x - x_1)$

Using $(5,6)$ and $m = \frac{1}{2}$ $\rightarrow$ $y - 6 = \frac{1}{2}(x - 5)$

$2(y - 6) = 1(x - 5)$

$2y - 12 = x - 5$

$\boxed{x - 2y + 7 = 0}$

b) When l_2 crosses the x-axis, $y = 0$

So, $x - 2(0) + 7 = 0$

$x = -7$

$\therefore$ A is at $(-7, 0)$

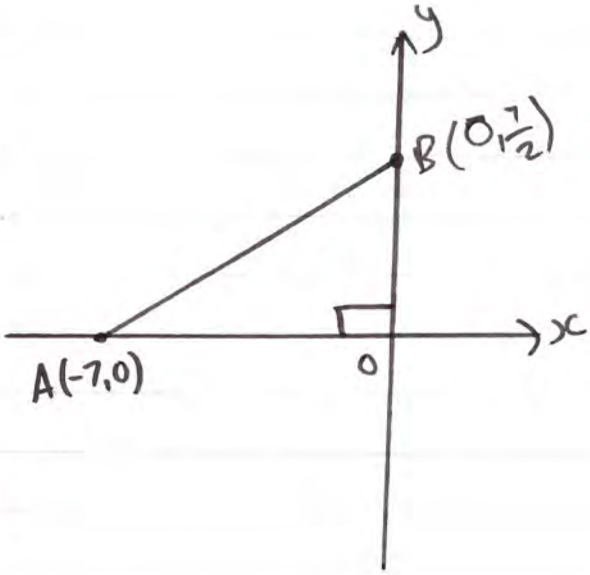
When l_2 crosses the y-axis, $x = 0$

So, $0 - 2y + 7 = 0$

$y = \frac{7}{2}$

$\therefore$ B is at $(0, \frac{7}{2})$

c) Length $OA = 7$ units
Length $OB = \frac{7}{2}$ units



Area $\Delta OAB = \frac{1}{2}bh$

$$= \frac{1}{2}(OA)(OB)$$

$$= \frac{1}{2}(7)\left(\frac{7}{2}\right)$$

$$= \frac{1}{2}\left(\frac{49}{2}\right)$$

$$= \boxed{\frac{49}{4} \text{ units}^2}$$

Q6 a) Let $f(x) = \frac{2}{x}$

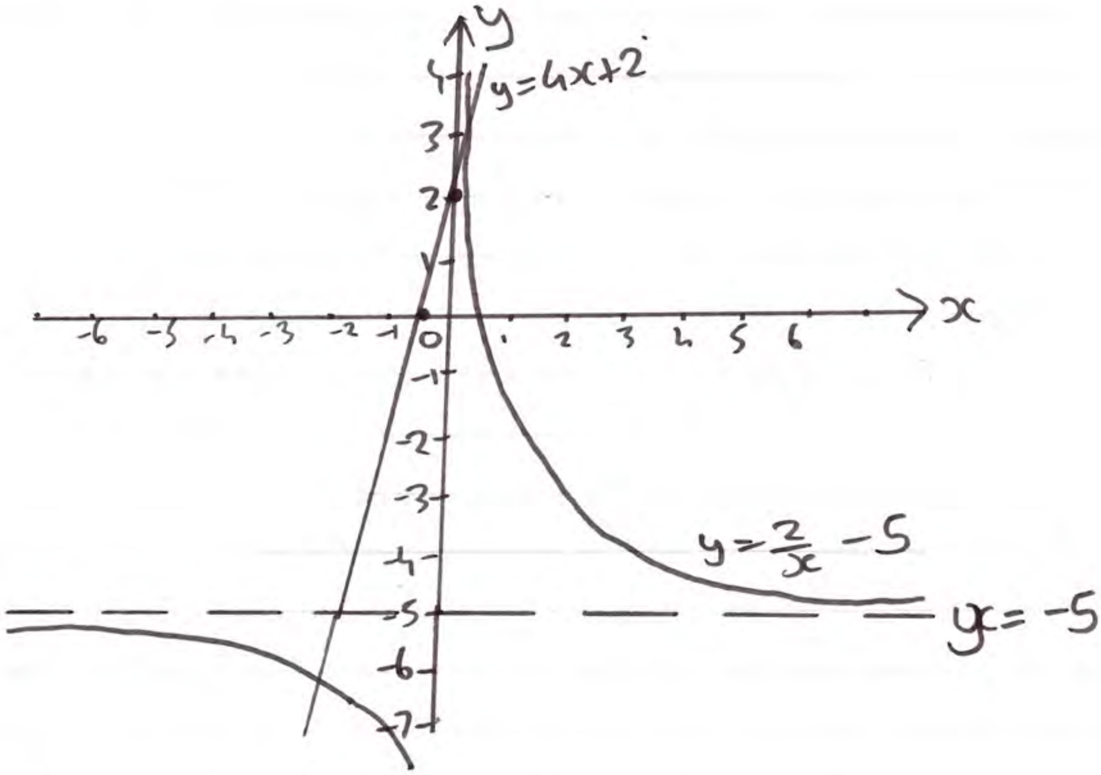
Then $\frac{2}{x} - 5$ is equal to $f(x) - 5$,

which is a transformation of -5 along the y-axis.

For line l, $y = 4x + 2$

When $\underline{x = 0, y = 2}$

When $y = 0$, $4x + 2 = 0$
$4x = -2$
$\underline{x = -\frac{1}{2}}$



b) $y = \frac{2}{x} - 5$ has asymptotes at :

$$\boxed{x = 0 \quad \text{and} \quad y = -5}$$

c) To find points of intersection, solve simultaneous equations:

$$y = \frac{2}{x} - 5 \quad ①$$

$$y = 4x+2 \quad ②$$

Substitute ① into ②:

$$\frac{2}{x} - 5 = 4x+2$$

$$\frac{2}{x} = 4x+7$$

$$4x+7 - \frac{2}{x} = 0$$

$$\frac{4x^2+7x-2}{x}=0$$

$$4x^2+7x-2=0$$

$$(4x-1)(x+2)=0$$

Either $x=\frac{1}{4}$ or $x=-2$

Substitute into ② for y:

When $x=\frac{1}{4}$, $y=4\left(\frac{1}{4}\right)+2 \underline{\underline{=3}}$

When $x=-2$, $y=4(-2)+2 \underline{\underline{=-6}}$

The coordinates of the points of intersection are:

$$\boxed{\left(\frac{1}{4},3\right) \text{ and } (-2,-6)}$$

Q7) First term, $a = 140$
Common difference, $d = 20$

a) $U_n = a+(n-1)d$

$U_{20} = 140+(20-1)20$

$U_{20} = 140+(19)20$

$\boxed{U_{20} = 520}$

b) $S_n = \frac{n}{2}(2a+(n-1)d)$

$S_{20} = \frac{20}{2}(2(140)+(20-1)20)$

$S_{20} = 10(280+(19)20)$

$S_{20} = 10(660)$

$\boxed{S_{20} = 6600}$

c) For Sian, first term, $a = 300$
common difference, $d = d$

$U_n = a+(n-1)d$

$700 = 300+(n-1)d$

$\underline{400 = (n-1)d}$

Also, $S_n = \frac{n}{2}(2a+(n-1)d)$

$8500 = \frac{n}{2}(2(300)+(n-1)d)$

$17000 = n(600+(n-1)d)$

$\underline{17000 = 600n + n(n-1)d}$

But $(n-1)d = 400$

$$\therefore 17000 = 600n + n(400)$$

$$17000 = 600n + 400n$$

$$1000n = 17000$$

$$\boxed{n = 17}$$

Q8) $$\frac{dy}{dx} = -x^3 + \frac{4x-5}{2x^3}, \quad x \neq 0$$

$$\frac{dy}{dx} = -x^3 + \frac{4x}{2x^3} - \frac{5}{2x^3}$$

$$\frac{dy}{dx} = -x^3 + 2x^{-2} - \frac{5}{2}x^{-3}$$

$$y = \int \left(-x^3 + 2x^{-2} - \frac{5}{2}x^{-3}\right) dx$$

$$y = -\frac{x^4}{4} + \frac{2x^{-1}}{-1} - \frac{5}{2} \cdot \frac{x^{-2}}{-2} + c$$

$$y = -\frac{x^4}{4} - 2x^{-1} - \frac{5x^{-2}}{-4} + c$$

$$y = -\frac{x^4}{4} - \frac{2}{x} + \frac{5}{4x^2} + c$$

Since $y=7$ at $x=1$, substitute in $x=1$ and $y=7$:

$$7 = \frac{-1^4}{4} - \frac{2}{1} + \frac{5}{4(1)^2} + c$$

$$7 = \frac{-1}{4} - 2 + \frac{5}{4} + c$$

$$c = 7 + \frac{1}{4} + 2 - \frac{5}{4}$$

$$\therefore c = 8$$

$$\boxed{y = \frac{-x^4}{4} - \frac{2}{x} + \frac{5}{4x^2} + 8}$$

Q9) $(k+3)x^2 + 6x + k = 5$ has two distinct real solutions for x.

a) If a quadratic has two distinct real solutions, then the discriminant is greater than 0.

$$\therefore b^2 - 4ac > 0$$

The equation $(k+3)x^2 + 6x + k = 5$, is also equal to $(k+3)x^2 + 6x + (k-5) = 0$, with $b = 6$, $a = (k+3)$, and $c = (k-5)$

$$b^2 - 4ac = 6^2 - (4)(k+3)(k-5)$$

$$\therefore 6^2 - 4(k+3)(k-5) > 0$$

$$36 - 4(k^2 - 2k - 15) > 0$$

$$36 - 4k^2 + 8k + 60 > 0$$

$$4k^2 - 8k - 96 < 0$$

$$\therefore \boxed{k^2 - 2k - 24 < 0}$$

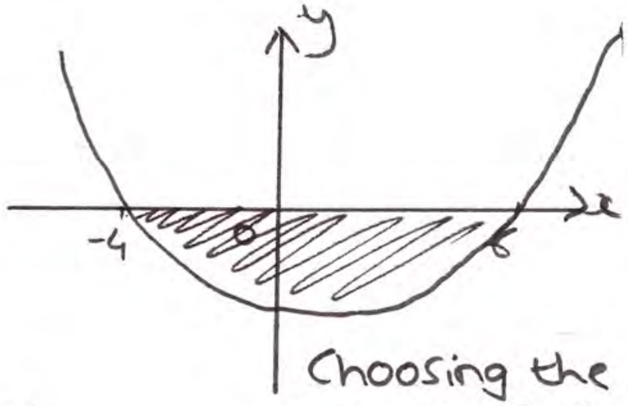
b) For $k^2 - 2k - 24 = 0$

$$(k-6)(k+4) = 0$$

Either $k=6$ or $k=-4$

Set of possible values of k:

$$\boxed{-4 < k < 6}$$



Choosing the values 'under' the x-axis, since $b^2-4ac<0$

Q10a) $4x^2 + 8x + 3 = 4\left(x^2 + 2x + \frac{3}{4}\right)$

$$= 4\left((x+1)^2 - 1^2 + \frac{3}{4}\right)$$

$$= \left(4\left((x+1)^2 - 1 + \frac{3}{4}\right)\right)$$

$$= 4(x+1)^2 - 4 + 3$$

$$= 4(x+1)^2 - 1$$

$$\therefore \boxed{a = 4, b = 1, c = -1}$$

b) $y = 4x^2 + 8x + 3$

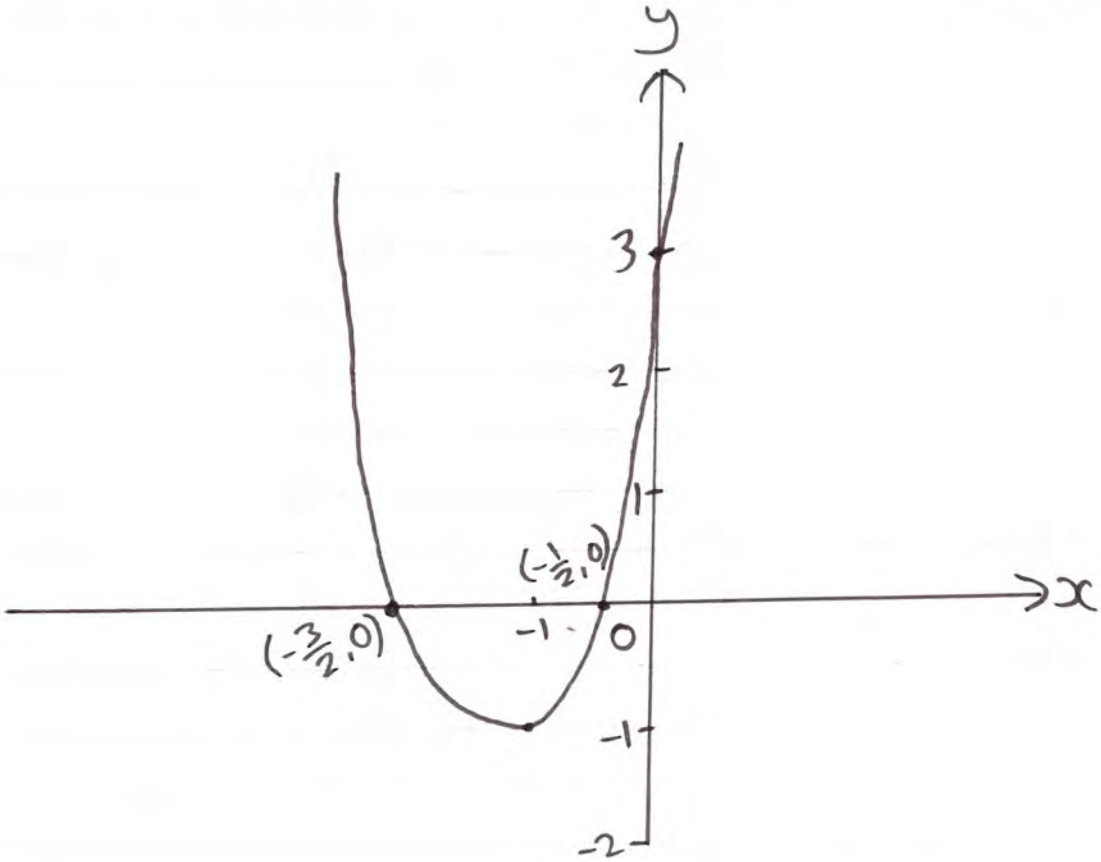
When $x = 0, y = 3$

When $y = 0$, $4x^2 + 8x + 3 = 0$

$(2x+3)(2x+1) = 0$

Either $x = \frac{-3}{2}$ or $x = \frac{-1}{2}$

Turning point from (a) is $(-1, -1)$



Q11a) $y = 2x - 8\sqrt{x} + 5, x \geq 0$

$y = 2x - 8x^{\frac{1}{2}} + 5$

$\frac{dy}{dx} = 2 - 4x^{-\frac{1}{2}}$

$= \boxed{2 - \frac{4}{\sqrt{x}}}$

b) When $x = \frac{1}{4}$, $y = 2\left(\frac{1}{4}\right) - 8\left(\sqrt{\frac{1}{4}}\right) + 5$

$= \frac{1}{2} - 8\left(\frac{1}{2}\right) + 5$

$= \frac{1}{2} - 4 + 5$

$= \underline{\frac{3}{2}}$

$\therefore$ P has coordinates $\left(\frac{1}{4}, \frac{3}{2}\right)$

When $x = \frac{1}{4}$, $\frac{dy}{dx} = 2 - \frac{4}{\sqrt{1/4}}$

$= 2 - \frac{4}{\frac{1}{2}}$

$= 2 - 8$

$= \underline{-6}$

$\therefore$ the gradient of the tangent at P is $\underline{-6.}$

Equation of tangent: $y - y_1 = m(x - x_1)$

Using $P(\frac{1}{4}, \frac{3}{2})$, and $m = -6$ $\longrightarrow$ $y - \frac{3}{2} = -6(x - \frac{1}{4})$

$$y - \frac{3}{2} = -6x + \frac{6}{4}$$

$$y = -6x + \frac{6}{4} + \frac{3}{2}$$

$$y = -6x + \frac{3}{2} + \frac{3}{2}$$

$$\boxed{y = -6x + 3}$$

c) $2x - 3y + 18 = 0$

$$3y = 2x + 18$$

$$y = \frac{2}{3}x + \frac{18}{3}$$

$$y = \frac{2}{3}x + 6$$

If the tangent to the curve is parallel to $2x - 3y + 18 = 0$ at point Q, then its gradient is also $\underline{\frac{2}{3}}$.

$$\therefore \frac{dy}{dx} = \frac{2}{3} = 2 - \frac{4}{\sqrt{x}}$$

$$\frac{2}{3} = 2 - \frac{4}{\sqrt{x}}$$

$$-\frac{4}{3} = -\frac{4}{\sqrt{x}}$$

$$\frac{4}{\sqrt{x}} = \frac{4}{3}$$

$$4 = \left(\frac{4}{3}\right)\sqrt{x}$$

$$\frac{4}{\left(\frac{4}{3}\right)} = \sqrt{x}$$

$$4 \times \frac{3}{4} = \sqrt{x}$$

$$3 = \sqrt{x}$$

$$\therefore \underline{x = 9}$$

When $x = 9$, $y = 2(9) - 8(\sqrt{9}) + 5$

$$= 18 - 24 + 5$$

$$\underline{= -1}$$

$\therefore$ the $\boxed{\text{coordinates of Q are } (9,-1)}$